

(a) Diffusion time $t_D \cong \frac{x^2}{4\alpha}$

Using $x \cong \frac{R}{2} = 0.05m$, $\alpha = \frac{K}{\rho c} = \frac{0.68}{1000 \times 4180} = 1.63 \times 10^{-7} m^2/s$

$$t_D \cong \frac{0.05^2}{4 \times 1.63 \times 10^{-7}} = 3,840 s = \underline{\underline{64 \text{ min}}}$$

This is comparable to “normal cooking times”, so the heat is diffusing throughout this time, and an isothermal model would not be right. In fact, we expect the outside to be better cooked than the inside, which sounds right.

(b) $M_{chicken} = \frac{4}{3} \pi R^3 \rho = \frac{4}{3} \pi \times 0.1^3 \times 1000 = 4.19kg$

$$M_{ch} c \frac{dT}{dt} = Q \quad (\text{heat input})$$

Take $c = 4180 \frac{J}{KgK}$, $Q = 1000W$, integrate

$$T - T_o = \frac{Q}{M_{ch} c} t$$

$$t_{cook} = \frac{M_{ch} c (T - T_o)}{Q} = \frac{4.19 \times 4180 \times (373 - 273)}{1000} = 1750s = \underline{\underline{29.2 \text{ min}}}$$

(c) In the Microwave case, t_{cook} is indeed proportional to chicken mass. But for the regular radiant oven, $t_D \sim R^2 \sim M_{ch}^{2/3}$. Doubling the mass increases the cooking time only by $2^{2/3} = 1.68$, so if you follow the cookbook guidelines, your turkey will be overdone.